Worksheet 2 Simplifying Boolean expressions
Answers

Task 1

1. X, Y and Z are Boolean variables which can be either TRUE or FALSE, represented
by 1 and 0.

Complete the following "rules" of Boolean algebra:

General rules

1. X ∧ 0 = 0
2. X ∧ 1 = X
3. X ∧ X = X
4. X ¬X = 0
5. X ∨ 0 = X
6. X ∨ 1 = 1
7. X ∨ X = X
8. X ∨ ¬X = 1
9. ¬X = X

Commutative rule

10. X ∧ Y = Y ∧ X
11. X ∨ Y = Y ∨ X

Associative rule

12. X ∧ (Y ∧ Z) = (X ∧ Y) ∧ Z
13. X ∨ (Y ∨ Z) = (X ∨ Y) ∨ Z

Distributive rule

14. X ∧ (Y ∨ Z) = (X ∧ Y) ∨ (X ∧ Z)
15. (X ∨ Y) ∧ (W ∨ Z) = (X ∧ W) ∨ (X ∧ Z) ∨ (Y ∧ W) ∨ (Y ∧ Z)

Absorption rules

16. X ∨ (X ∧ Y) = X
17. X ∧ (X ∨ Y) = X

2. Write down de Morgan’s first and second laws:

   ¬(A ˅ B) = ¬A ˄ ¬B

   ¬(A ˄ B) = ¬A ˅ ¬B

3. Use de Morgan’s Laws and the rules of Boolean algebra to simplify the following expressions, stating which rule you use at each step.

   (a) X ˄ Y ˅ X ˄ (Y ˅ Z)

   = (X ˄ Y) ˅ (X ˄ Y) ˅ (X ˄ Z) Distributive Rule

   = (X ˄ Y) ˅ (X ˄ Z) Rule 7

   (b) (X ˅ Y) ˄ (¬X ˅ ¬Y)

   = $(XX)(XY)(YX)(YY)$ Distributive Rule

   = $0(XY)(YX)0$ Rule 4

   = $XYYX$ Rule 5

   (c) $X(XY)$

   = XXY (de Morgan’s Law)

   ¿$1\ Y$ Rule 8

   ¿1 Rule 6

   (d) (X Y) (X Z)

   = (XX) (XZ) (YX) (YZ) Distributive Law

   = X (XZ) (YX) (YZ) Rule 3

   = X (YX) (YZ) Absorption rule

   = X YZ Absorption rule

   Note that “multiplying out” the brackets (X Y) (X Z) is very much easier in the alternative notation using . for AND and + for OR, giving

   (X + Y).(X + Z) = (X.X + X.Z + Y.X + Y.Z) as in maths notation

4. Complete the truth table to show that $A\ A\ B = A\ B$

A	B	¬A	¬A ˄ B	A ˅ ¬A ˄ B	A ˅ B
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

Task 2

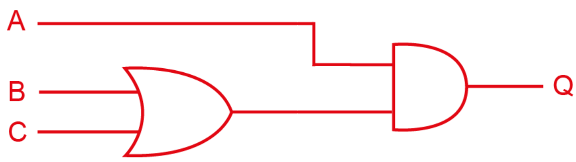
5. Simplify the expression A B A (B C)

A AB   (B C)   commutative rule

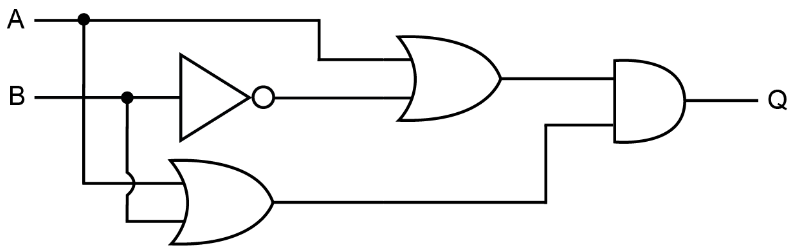
A (A B) (B C))

A(B C)   absorption rule

Draw a logic circuit representing the simplified expression, using only 2 gates.



6. (a) Write the Boolean expression representing the logic circuit below.



(A ¬B) (A B)

(b) Complete the truth table to prove the Absorption rules:

X ˅ (X ˄ Y) = X

X ˄ (X ˅ Y) = X

X	Y	(X ˄ Y)	(X ˅ Y)	X ˅ (X ˄ Y)	X ˄ (X ˅ Y)
0	0	0	0	0	0
0	1	0	1	0	0
1	0	0	1	1	1
1	1	1	1	1	1

(c) Simplify the expression.

A A A B ˅ A ¬B ˅ B ¬B   (Tip: Use the Absorption rule)

In the alternative notation, (AND) is the same as . and ˅ (OR) is the same as +.

As in Maths, given for example A . B + A . C, you can insert brackets to get (A . B) + (A . C), but you cannot insert brackets to the terms either side of the + to give A . (B + A) . C

Start by bracketing the expressions involving AND on each side of the OR operators

= (A A) (A B) ˅ (A ¬B) ˅ (B ¬B)

Next, note that (A A) = A, (Rule 3) and (B ¬B) = 0 (Rule 4)

So we get

A (A B) ˅ (A ¬B) ˅ 0

A (A B) ˅ (A ¬B)   Rule 5

= A ˅ (A ¬B)   Using the Absorption rule

= A   Using the Absorption rule again

(d) With reference to the above example, explain why de Morgan's Laws and the rules of Boolean algebra have a huge commercial significance in the manufacture of computers.

In the above example, the output Q has been reduced to a single input A, saving on the manufacture of four unnecessary logic gates. This saves both manufacturing costs and processing time.